AP Calculus BC

Unit 3 – Advanced Differentiation Techniques

AP Calculus BC – Worksheet 14 **Derivatives of Exponential and Logarithmic Functions**

Find $\frac{dy}{dx}$ for each of the following.

1. $y = x^2 \ln x$

2. $y = \sin(2x) + 2^{\sin x}$

3. $y = e^{2x}$

4. $g(x) = \log_9 \left(6x^4 + 3\right)^5$

5. $y = 5^{3x}$

6. $f(x) = 5^{x^3 - 7}$

7. $f(x) = \ln\left(4x^3 + \sec x\right)$

8. $y = \sin(\ln x)$

9. $y = \left(x^2 + 1\right)^3 \left(4x + 3\right)^5$

10. $f(x) = e^{\tan x}$

11. $y = x^5 3^{-3x}$

12. $g(x) = \sec\left(5^{2x}\right) + \ln\sqrt{4x + 2}$

13. Write an equation for the tangent and normal lines to $y = xe^{-x}$ when $x = 1$.

14. At what point on the graph of $y = 4^x + 3$ is the tangent line parallel to the line $y = 2x - 9$?

1	Use implicit differentiation to find $\frac{dy}{dx}$ for $x = \sec y$.
2	For $2x^2 - y^2 = 1$, find: a) $\frac{dy}{dx}$ b) $\frac{d^2y}{dx^2}$ and simplify in terms of *x* and *y*.
3	For $y^2 = 9x^2 + 4x$, find: a) $\frac{dy}{dx}$ b) $\frac{d^2y}{dx^2}$ and simplify in terms of *x* and *y*.
4	Use the curve $x^2 - 4xy + y^2 = -6$. Show that $\frac{dy}{dx} = \frac{4y - 2x}{2y - 4x}$.
5	For $x^2 + y^2 = 26$, determine the equations of the tangent lines when $x = -1$.
6	Find the slope of the tangent line to the curve $(x-3)^2 + (y-4)^2 = 5$ at the point $(5,5)$.
7	Find the equations of the lines that are tangent and normal to the curve $x^2y^2 = 16$ at $(-1,4)$.

1. Consider the curve defined $xy^2 - 2x^3 = 2$ for $y \geq 0$.

   a) Show that $\dfrac{dy}{dx} = \dfrac{6x^2 - y^2}{2xy}$.

   b) Write an equation for the line tangent to the curve at the point $(1, 2)$.

   c) Find the x-coordinate of the point P at which the line tangent to the curve at P is horizontal.

   d) Find the value of $\dfrac{d^2y}{dx^2}$ at the point $(1, 2)$.

2. Consider the curve defined by $y^2 - x^2 y = 6$ for $y > 0$.

   a) Show that $\dfrac{dy}{dx} = \dfrac{2xy}{2y - x^2}$

   b) Write an equation for the line tangent to the curve at the point $(1, 3)$.

   c) Show that there is a point P with x-coordinate 0 at which the line tangent to the curve P is horizontal. Find the y-coordinate of point P.

   d) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (c).

AP Calculus BC - Worksheet 17 **Derivatives of Inverse Trig Functions and Implicit Differentiation**

Find the derivatives

1) $y=\sin^{-1}(5x)$	2) $y=\csc^{-1}(4x^5)$	3) $y=\arctan(e^{2x})$
4) $y=\cot^{-1}(3x^2-1)$	5) $y=\arcsin\left(\frac{1}{x}\right)$	6) $y=\operatorname{arcsec}(x^3)$

Find the equation of the tangent line to the curve at the given value of *x*.

7) $y=\arcsin x$; $x=\frac{\sqrt{2}}{2}$	8) $f(x)=\cos^{-1}(4x)$; $x=\frac{\sqrt{3}}{8}$
9) $f(x)=\arctan x$; $x=1$	10) $f(x)=\sin^{-1}(5x)$; $x=-\frac{\sqrt{3}}{10}$

11) Let $g(x)=(\arccos x^2)^5$, then $g'(x)=$
12) If $\lim_{x\to a}\frac{\arccos x-\arccos a}{x-a}=3$, find the value of a.
13) If $\arctan y=\ln x$, find $\frac{dy}{dx}$.
14) If $y=e^x(\sec^{-1}x)$, find $\frac{dy}{dx}$.
15) If $y^2-8y+x^2=5$, find $\frac{dy}{dx}$.

AP Calculus BC – Worksheet 18 **Derivative of Inverse Functions**

1. Let *f* be the function defined by $f(x) = x^3 + 7x + 2$. If $g(x) = f^{-1}(x)$, evaluate the following:
   a) $f(1)$
   b) $f'(1)$
   c) $g(10)$
   d) $g'(10)$?

2) Let *f* be the function defined by $f(x) = x^5 + 3x^3 + 7x + 2$. If $g(x) = f^{-1}(x)$ and $f(1) = 13$, what is the value of $g'(13)$?

3) Let *f* be the function defined by $f(x) = 7(x+1)^3 + \sin^3 x$. If $g(x) = f^{-1}(x)$ and $f(0) = 7$, what is the value of $g'(7)$?

4) Let *f* be the function defined by $f(x) = x^7 + 2x + 9$. If $g(x) = f^{-1}(x)$, find $g'(12)$.

5) Let *f* be the function defined by $f(x) = x^3 + x - 8$. If $g(x) = f^{-1}(x)$, find $g'(-6)$.

6) The functions *f* and *g* are differentiable. Given that $g(x) = f^{-1}(x)$, $f(1) = 3$, and $f'(1) = -5$, find $g'(3)$

7) The functions *f* and *g* are differentiable. Given that $g(x) = f^{-1}(x)$, $f(2) = 4$, $f(4) = -6$, $f'(2) = 7$, and $f'(4) = 11$, find $g'(4)$.

8) Find $\dfrac{d^2y}{dx^2}$ for $y = \arcsin(3x+2)$

9) Given the function $y = \arctan(\cos x)$. Find the value of $f''\left(\dfrac{\pi}{3}\right)$.

Find the derivative of each function.

1) $f(x) = 2\sin x \cos x$	2) $s = \cot\frac{2}{t}$	3) $r = \sec(1+3\theta)$	4) $y = \ln\sqrt{x}$
5) $y = e^{(1+\ln x)}$	6) $r = \log_2(\theta^2)$	7) $y = x^{\ln x}$	8) $f(x) = (\sin x)^x$
9) $f(x) = x\ln x$	10) $xy + 2x + 3y = 1$	11) $y^2 = \frac{x}{x+1}$	12) $\sqrt{xy} = 1$

13	Find $\frac{d^2y}{dx^2}$ for $x^3 + y^3 = 1$.
14	Find $\frac{d^2y}{dx^2}$ for $f(x) = xe^{\sin x}$.
15	Find the equation for the (a) tangent and (b) normal line to the graph of $f(x) = \sqrt{x^2 - 2x}$ when $x = 3$.
16	Find the equation for the (a) tangent and (b) normal line to the graph of $x + \sqrt{xy} = 6$ at $(4,1)$.
17	See below

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Working with Numerical Values Suppose that a function f and its first derivative have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$f'(x)$
0	9	-2
1	-3	1/5

Find the first derivative of the following combinations at the given value of x.

a) $\sqrt{x}f(x)$, $x = 1$

b) $f(1 - 5\tan x)$, $x = 0$

c) $\frac{f(x)}{2 + \cos x}$, $x = 0$

AP Calculus BC – Worksheet 20 **Derivatives Review**

I. Differentiate

1. $y=\ln\left(5x^4\right)$

2. $y=e^{-4x}$

3. $y=7^{3x^2+4x}$

4. $f(x)=\sin x\cos x$

5. $y=x^{5x}$

6. $y=\left(x^3+2\right)^4\left(\cot x-2x\right)^5$

7. $y=\sin\left(3x-4\right)$

8. $w(x)=\tan^2\left(\ln(1+x)\right)$

9. $h(x)=\ln\left(\sec\sqrt{x}\right)$

10. $y=\arccos\left(5x^3+4x\right)$

11. $y=\cos\left(\ln 3x\right)$

12. $f(x)=\tan^3\left(4x^6-2x\right)$

13. $f(x)=\csc\left(x^6\right)e^{-5x}$

14. $y=\ln\left(x^2+5x\right)$

II. Applications

15. Given: $y=\sin^2 x$. Write the equations of the tangent and normal lines to the graph where $x=\dfrac{\pi}{6}$.

16. Given: $f(x)=\sin^2(x)$ and $g(x)=x^2-5$. Let $K(x)=g\left(f(x)\right)$.

   a. $K'(x)$

   b. Find $K'\left(\dfrac{\pi}{4}\right)$.

17. Given: $x^2+y^3=1$ find $\dfrac{d^2y}{dx^2}$ at (3, -2).

A graphing calculator is required for this question.

x	–3	0	3	4
$f(x)$	5	–1	2	7
$f'(x)$	–2	4	0	1

1. The table above gives values of a twice-differential function f and its first derivative f' for selected values of *x*. Let g be the function defined by $g(x) = f(2x - x^2)$.

   (a) What is the value of $g'(-1)$?

   (b) It is known that $g''(0) = 0$. What is the value of $f''(0)$?

   (c) Is there a value *c*, for $0 < c < 3$, such that $g(c) = 2$? Justify your answer.

   (d) Let h be the function with the first derivative given by $h'(x) = 4xe^x$. At what value of *x* in the interval $0 \le x \le 4$ does the instantaneous rate of change of h equal the average rate of change of *f* over the interval $0 \le x \le 4$?

No calculator is allowed for this question.

2. Consider the curve given by the equation $2xy + y^2 = 8$ for $y > 0$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{-y}{x+y}$.

   (b) Write an equation for the line tangent to the curve at the point $(1, 2)$.

   (c) Evaluate $\dfrac{d^2y}{dx^2}$ at the point $(1, 2)$.

   (d) The points $(1, 2)$ and $\left(\dfrac{7}{2}, 1\right)$ are on the curve. Find the value of $\left(y^{-1}\right)'(1)$.

No calculator is allowed for this question.

x	-3	2	3	8
$f(x)$	-9	4	2	6
$f'(x)$	$-\frac{7}{2}$	$\frac{3}{2}$	$-\frac{2}{5}$	$\frac{1}{3}$

3. The table above gives values of a differentiable function f and its derivative for selected values of x.

   (a) Let g be the function defined by $g(x) = \dfrac{\ln x}{f(x^3)}$. Find $g'(2)$.

   (b) Let h be the function defined by $f(f(-3x))$. Find $h'(-1)$.

   (c) Let k be the function defined by $k(x) = f(x) \cdot \arctan\left(\dfrac{x}{3}\right)$. Find $k'(-3)$.